

Lewis Carroll in Numberland

His Fantastical Mathematical Logical Life

An Agony in Eight Fits

ROBIN WILSON

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Introduction

From Gryphons to Gravity

"Begin at the beginning," the King said, very gravely,
"and go on till you come to the end: then stop."

As you might expect from a lecturer in mathematics, Lewis Carroll's books for children are brimming with mathematical allusions — arithmetical, geometrical, logical and mechanical. This is the world of mock turtles and maps, gryphons and gravity, Humpty Dumpty and handkerchiefs — recast here in dramatic form in eight scenes.

Scene 1: The Mock Turtle's Education

In *Alice's Adventures in Wonderland* (1865), Alice is introduced to the Gryphon, who leads her to a rocky seashore. There they encounter the Mock Turtle, who looks at them with large eyes full of tears.

Mock Turtle: Once I was a real turtle.

Gryphon: Hjckrrrh!

Mock Turtle: When we were little, we went to school in the sea. The master was an old turtle — we used to call him

Tortoise —

Alice: Why did you call him Tortoise, if he wasn't one?

Mock Turtle: We called him Tortoise because he taught us. Really you are very dull!

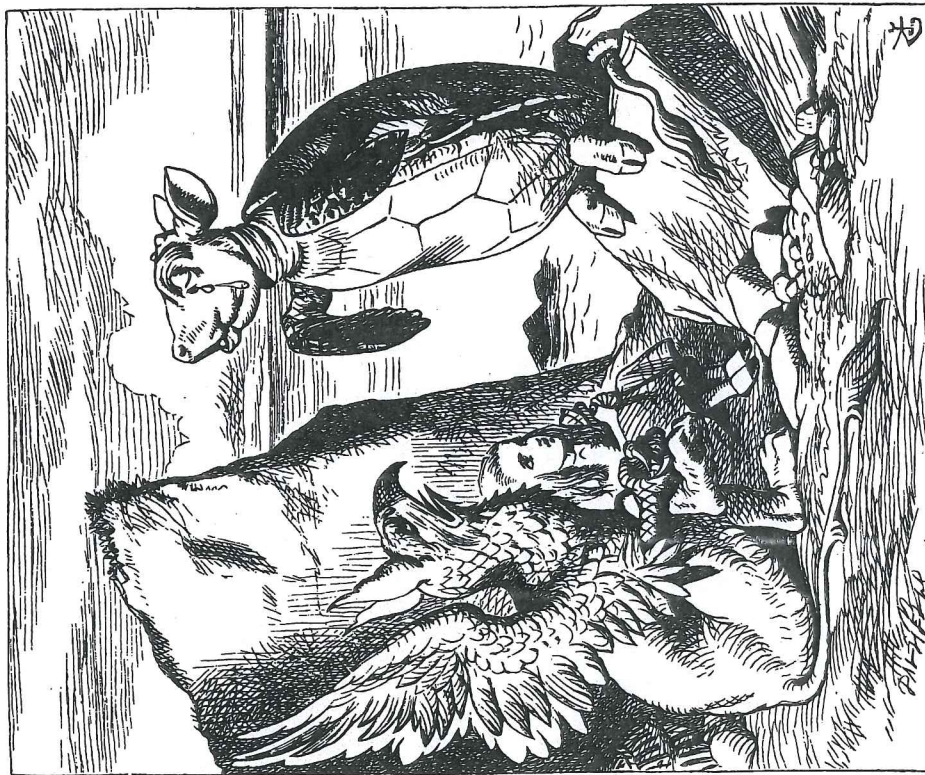
Gryphon: You ought to be ashamed of yourself for asking such a simple question.

Mock Turtle: Yes, we went to school in the sea. I only took the regular course.

Alice: What was that?

Mock Turtle: Reeling and Writhing, of course, to begin with; and then the different branches of Arithmetic — Ambition, Distraction, Uglification and Derision.

Alice: I never heard of 'Uglification'. What is it?



The Mock Turtle tells Alice his sad story

Gryphon: Never heard of uglifying! You know what to beauty is, I suppose?

Alice: Yes: it means — to — make — anything — prettier.

Gryphon: Well, then, if you don't know what to uglify is, you *are* a simpleton.

Alice: And how many hours a day did you do lessons?

Mock Turtle: Ten hours the first day, nine hours the next, and

so on.

Alice: What a curious plan!

Gryphon: That's the reason they're called lessons — because they lessen from day to day.

Alice: Then the eleventh day must have been a holiday.

Mock Turtle: Of course it was.

Alice: And how did you manage on the twelfth?

Gryphon: That's enough about lessons.

Scene 2: Humpty Dumpty's Cravat

In Lewis Carroll's second *Alice* book, *Through the Looking-Glass* (1871), Alice encounters the argumentative Humpty Dumpty, a stickler for the meaning of words, for whom a simple arithmetical calculation proves to be rather a challenge.

Humpty: Tell me your name and your business.

Alice: My *name* is Alice, but —

Humpty: It's a stupid name enough! What does it mean?

Alice: *Must* a name mean something?

Humpty: Of course it must: *my* name means the shape I am — and a good handsome shape it is, too. With a name like yours, you might be any shape, almost. How old did you say you were?

Alice: Seven years and six months.

Humpty: Wrong! You never said a word like it!

Alice: I thought you meant 'How old *are* you?'

Humpty: If I'd meant that, I'd have said it.

Alice: (*after a pause*) What a beautiful belt you've got on! At least, a beautiful cravat — no, a belt, I mean — I beg your pardon!

Humpty: It's a cravat, child, and a beautiful one, as you say. It's a present from the White King and Queen. They gave it me — for an un-birthday present.



Humpty Dumpty sat on a wall

Alice: I beg your pardon?

Humpty: I'm not offended.

Alice: I mean, what is an un-birthday present?

Humpty: A present given when it isn't your birthday, of course.

Alice: I like birthday presents best.

Humpty: You don't know what you're talking about! How many days are there in a year?

Alice: Three hundred and sixty-five.

Humpty: And how many birthdays have you?

Alice: One.

Humpty: And if you take one from three hundred and sixty-five, what remains?

Alice: Three hundred and sixty-four, of course.

Humpty: I'd rather see that done on paper.

Alice: Three hundred and sixty-five . . .

minus one . . .

is three hundred and sixty-four.

Humpty: That seems to be done right —

Alice: You're holding it upside down!

Humpty: To be sure I was! I thought it looked a little queer. As I was saying, that *seems* to be done right — though I haven't time to look over it thoroughly right now — and that shows that there are three hundred and sixty-four days when you might get un-birthday presents —

Alice: Certainly.

Humpty: And only *one* for birthday presents, you know. There's glory for you!

Alice: I don't know what you mean by 'glory'.

Humpty: Of course you don't — till I tell you. I meant 'there's a nice knock-down argument for you!'

Alice: But 'glory' doesn't mean 'a nice knock-down argument'.

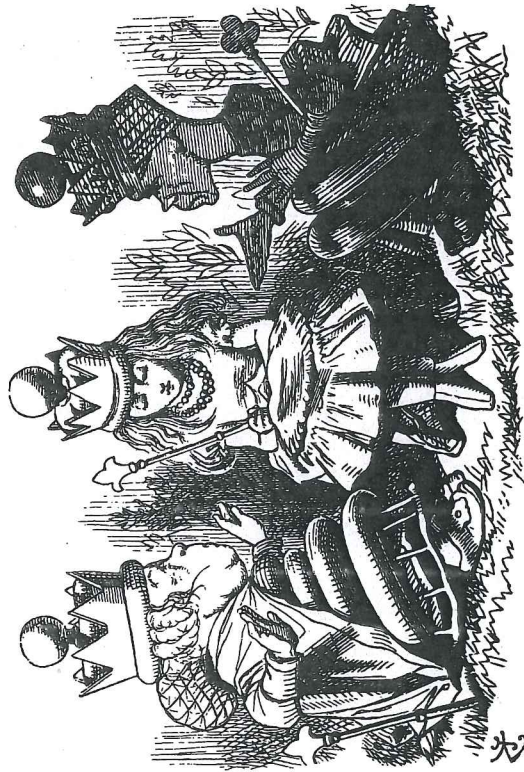
Humpty: When I use a word, it means just what I choose it to mean — neither more nor less.

Scene 3: Alice's Examination

When Alice finally reaches the Eighth Square on the looking-glass chessboard, she expects to become Queen — but first she must be interrogated by the Red Queen and the White Queen.

Red Queen: You ca'n't be a queen, you know, till you've passed the proper examination. And the sooner we begin it, the better.

White Queen: Can you do Addition? What's one and one and one and one and one and one and one and one and one?



Alice is examined by the White Queen and the Red Queen

Alice: I don't know. I lost count.

Red Queen: She ca'n't do Addition. Can you do Subtraction? Take nine from eight.

Alice: Nine from eight I ca'n't, you know: but —

White Queen: She ca'n't do Subtraction. Can you do Division? Divide a loaf by a knife. What's the answer to *that*?

Alice: I suppose —

Red Queen: Bread-and-butter, of course. Try another Subtraction sum. Take a bone from a dog: what remains?

Alice: The bone wouldn't remain, of course, if I took it — and the dog wouldn't remain: it would come to bite me — and I'm sure I shouldn't remain!

Red Queen: Then you think nothing would remain?

Alice: I think that's the answer.

Red Queen: Wrong, as usual. The dog's temper would remain.

Alice: But I don't see how —

Red Queen: Why, look here! The dog would lose its temper, wouldn't it?

Alice: Perhaps it would.

Red Queen: Then if the dog went away, its temper would remain!

Both Queens: She ca'n't do sums a *bit*!

Scene 4: What's in a Name?

Logical and philosophical absurdities permeate the *Alice* books — such as the Cheshire Cat's celebrated grin in *Alice's Adventures in Wonderland*:

"All right," said the Cat; and this time it vanished quite slowly, beginning with the end of the tail, and ending with the grin, which remained some time after the rest of it had gone.

"Well! I've often seen a cat without a grin," thought Alice; "but a grin without a cat! It's the most curious thing I ever saw in all my life!"

In *Through the Looking-Glass*, the White Queen challenges Alice about the nature of belief and the impossible:

White Queen: Let's consider your age to begin with — how old are you?

Alice: I'm seven and a half exactly.



White Queen: You needn't say 'exactly': I can believe it without that. Now I'll give *you* something to believe. I'm just one hundred and one, five months and a day.

Alice: I can't believe *that!*

White Queen: Ca'n't you? Try again: draw a long breath, and shut your eyes.

Alice: There's no use trying; one *ca'n't* believe impossible things.

White Queen: I daresay you haven't had much practice. When I was your age, I always did it for half-an-hour a day. Why, sometimes I've believed as many as six impossible things before breakfast.

After her meeting with Humpty Dumpty, Alice comes across the White King, who is busily trying to protect his crown from the Lion and the Unicorn:

White King: I've sent them all! Did you happen to meet any soldiers, my dear, as you came through the wood?

Alice: Yes, I did: several thousand, I should think.

White King: Four thousand two hundred and seven, that's the exact number. I couldn't send all the horses, you know, because two of them are wanted in the game. And I haven't sent the two

Messengers, either. They're both gone to the town. Just look along the road, and tell me if you can see either of them.

Alice: I see nobody on the road.

White King: I only wish I had such eyes. To be able to see Nobody! And at that distance too! Why, it's as much as I can do to see real people, by this light!

Once Haigha, the Messenger, arrives, he is quizzed in a similar vein:

White King: Who did you pass on the road?

Haigha: Nobody.

White King: Quite right: this young lady saw him too. So of course Nobody walks slower than you.



Alice meets the White Knight

The result we proceed to divide, as you see,
By Nine Hundred and Ninety and Two:
Then subtract Seventeen, and the answer must be
Exactly and perfectly true."

The arithmetic described in this verse is straightforward. In trying to explain to the Beaver why $2 + 1 = 3$, the Butcher starts with 3, adds 7 and 10, and multiplies by $1000 - 8$ (which is 992). He then divides by 992 and subtracts 17, taking him back to where he started — namely, 3:

$$\frac{(3 + 7 + 10) \times (1000 - 8)}{992} - 17 = 3$$

In fact, any number other than 3 would have done equally well — the Butcher must always end with the number he started with.

Scene 6: Map-making

Earlier, in *Fit the Second of The Hunting of the Snark*, the Bellman provides a map for his crew of Snark hunters to use:

The Bellman himself they all praised to the skies —
Such a carriage, such ease and such grace!
Such solemnity, too! One could see he was wise,
The moment one looked in his face.

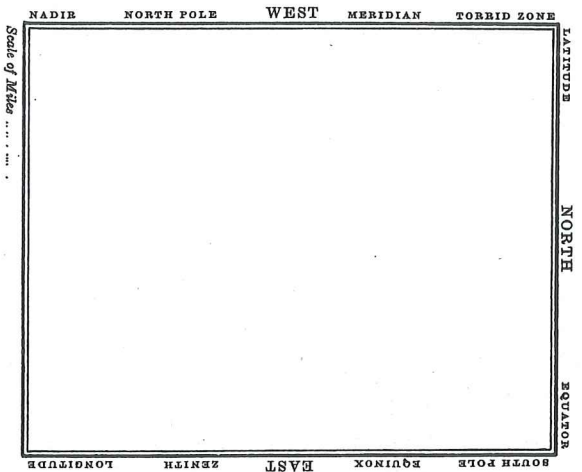
He had bought a large map representing the sea,
Without the least vestige of land;
And the crew were much pleased when they found it to be
A map they could all understand.

"What's the good of Mercator's North Poles and Equators,
Tropics, Zones and Meridian Lines?"
So the Bellman would cry: and the crew would reply,
"They are merely conventional signs!"

"Other maps are such shapes, with their islands and capes!
But we've got our brave Captain to thank"
(So the crew would protest) "that he's bought us the best —
A perfect and absolute blank!"



The Butcher instructs the Beaver



The Bellman's ocean chart

A different type of map is described in *Sylvie and Bruno Concluded* (1893), Lewis Carroll's last novel for children and the sequel to *Sylvie and Bruno* (1889). In this scene, the book's narrator (Myself) and the fairy children Sylvie and Bruno are listening to Mein Herr, a grand old German gentleman with a long beard, who explains to us how maps are constructed in his own country:

Myself: What a useful thing a pocket-map is!
 Mein Herr: That's another thing we've learned from *your* Nation, map-making. But we've carried it much further than *you*. What do you consider the *largest* map that would be really useful?

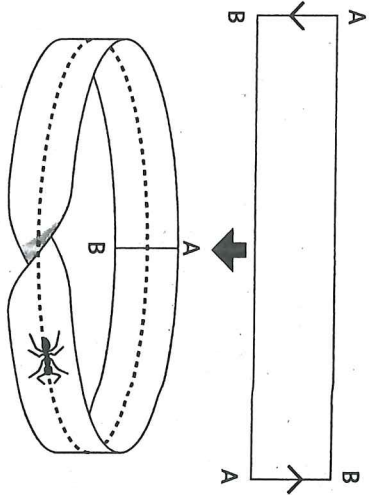
Myself: About: six inches to the mile.
 Mein Herr: Only *six inches*! We very soon got to six *yards* to the mile. Then we tried a *hundred* yards to the mile. And then

came the grandest idea of all! We actually made a map of the country, on the scale of *a mile to the mile*!
 Myself: Have you used it much?
 Mein Herr: It has never been spread out, yet: the farmers objected: they said it would cover the whole country, and shut out the sunlight! So we now use the country itself, as its own map, and I assure you it does nearly as well.

Scene 7: Fortunatus's Purse

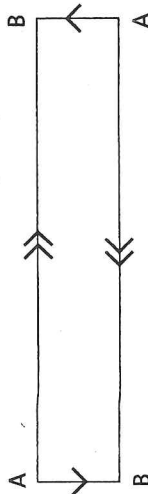
In *Sylvie and Bruno Concluded*, Carroll's ability to illustrate mathematical ideas in a painless and picturesque way is shown in the construction of *Fortunatus's Purse* from three handkerchiefs. This purse has no inside or outside, and so can be considered to contain the entire wealth of the world.

The passage includes a description of a 'Paper Ring', or *Möbius band*, named after the nineteenth-century German mathematician and astronomer August Ferdinand Möbius. This can be made from a rectangular strip of paper by twisting one end through 180 degrees and then gluing the two ends together, as pictured here. The resulting object has just one side and just one edge: this means that an insect could travel from any point on it to any other point without leaving the surface or going over the edge.



Constructing a Möbius band

An extension of this idea is to start with a rectangular strip and try to glue *both* pairs of opposite sides in opposing directions. This cannot be done in our three-dimensional world, however.



Constructing Fortunatus's purse

The resulting object — Fortunatus's Purse — has the form of a mathematical object called a *projective plane*. Since it cannot exist in three dimensions, the description that follows ceases just before the task becomes impossible.

We are in a shady nook where afternoon tea is being enjoyed. Lady Muriel is sewing, while her father (the Earl of Ainslie) and the narrator look on. Along comes the venerable Mein Herr.

Mein Herr: Hemming pocket-handkerchiefs? So *that* is what the English miladies occupy themselves with, is it?

Myself: It is the one accomplishment in which Man has never yet rivalled Woman!

Mein Herr: You have heard of Fortunatus's Purse, Miladi? Ah, so! Would you be surprised to hear that, with three of these leetle handkerchiefs, you shall make the Purse of Fortunatus quite soon, quite easily?

Lady Muriel: Shall I indeed? Please tell me how, Mein Herr! I'll make one before I touch another drop of tea!

Mein Herr: You shall first join together these upper corners, the right to the right, the left to the left; and the opening between them shall be the *mouth* of the Purse.

Lady Muriel: Now if I sew the other three edges together, the bag is complete?

Mein Herr: Not so, Miladi: the *lower* edges shall *first* be joined — ah, not so! Turn one of them over, and join the *right* lower corner of the one to the *left* lower corner of the other, and sew the lower edges together in what you would call *the wrong way*.

Lady Muriel: I see! And a very twisted, uncomfortable, uncanny-looking bag it makes! But the *moral* is a lovely one. Unlimited wealth can only be obtained by doing things *in the wrong way*! And how are we to join up these mysterious — no, I mean *this* mysterious opening? Yes, it is one opening — I thought it was *two*, at first.

Mein Herr: You have seen the puzzle of the Paper Ring? Where you take a slip of paper, and join its ends together, first twisting one, so as to join the *upper* corner of *one* end to the *lower* corner of the *other*?

The Earl: I saw one made, only yesterday. Muriel, my child, were you not making one, to amuse those children you had to tea?

Lady Muriel: Yes, I know that Puzzle. The Ring has only *one* surface, and only *one* edge. It's very mysterious!

Myself: The *bag* is just like that, isn't it? Is not the *outer* surface of one side of it continuous with the *inner* surface of the other side?



Mein Herr manipulates the handkerchiefs

Lady Muriel: So it is! Only it *isn't* a bag, just yet. How shall we fill up this opening, Mein Herr?

Mein Herr: Thus! The edge of the opening consists of *four* handkerchief-edges, and you can trace it continuously, round and round the opening: down the right edge of *one* handkerchief, up the left edge of the *other*, and then down the left edge of the *one*, and up the right edge of the *other*!

Lady Muriel: So you can! And that *proves* it to be only *one* opening!

Mein Herr: Now, this *third* handkerchief has *also* four edges, which you can trace continuously round and round: all you need do is to join its four edges to the four edges of the opening. The Purse is then complete, and its outer surface —

Lady Muriel: I see! Its *outer* surface will be continuous with its *inner* surface! But it will take time. I'll sew it up after tea. But why do you call it Fortunatus's Purse, Mein Herr?

Mein Herr: Don't you see, my child — I should say Miladi? Whatever is *inside* that Purse, is *outside* it; and whatever is *outside* it, is *inside* it. So you have all the wealth of the world in that leetle Purse!

Lady Muriel: I'll certainly sew the third handkerchief in — *some* time, but I wo'n't take up your time by trying it now.

Scene 8: A Question of Gravity

Still in our shady nook, Mein Herr reminisces about various inventions to be seen in his country, including a train that runs entirely by gravity.

Lady Muriel: *Please* tell us some more wonderful things!

Mein Herr: They run their railway-trains without any engines — nothing is needed but machinery to *stop* them with. Is *that* wonderful enough, Miladi?

Myself: But where does the *force* come from?

Mein Herr: They use the force of *gravity*. It is a force known also in *your* country, I believe?

The Earl: But that would need a railway going *down-hill*. You ca'n't have *all* your railways going down-hill!

Mein Herr: They *all* do.

The Earl: Not from *both* ends?

Mein Herr: From *both* ends.

The Earl: Then I give it up!

Lady Muriel: Can you explain the process?

Mein Herr: Easily. Each railway is in a long tunnel, perfectly straight: so of course the *middle* of it is nearer the centre of the globe than the two ends: so every train runs half-way *down-hill*, and that gives it force enough to run the *other* half *up-hill*.

Lady Muriel: Thank you. I understand that perfectly. But the velocity in the *middle* of the tunnel must be something *fearful*!

Gravity fascinated Lewis Carroll. *Alice's Adventures in Wonderland* commences with Alice tumbling down a deep rabbit-hole and wondering to herself how far she had fallen:

I wonder how many miles I've fallen by this time? I must be getting somewhere near the centre of the earth. Let me see: that would be four thousand miles down, I think . . . I wonder if I shall fall right *through* the earth! How funny it'll seem to come out among the people that walk with their heads downwards! The Antipathies, I think . . .

While descending, she takes a jar labelled ORANGE MARMALADE from a shelf and finds, to her great disappointment, that it is empty. She decides not to drop it for fear of killing anyone underneath, forgetting that it would remain suspended in front of her as she continued to fall.

This idea is developed further in *Sylvie and Bruno*, where Lady Muriel, her father the Earl and the narrator (Myself) are in conversation with a young doctor called Arthur. The narrator has just insisted on taking a cup of tea across the room to the Earl, and the conversation soon turns to the problem of drinking tea inside a falling house:

Lady Muriel: How convenient it would be if cups of tea had no weight at all! Then perhaps ladies would *sometimes* be permitted to carry them for short distances!

Arthur: One can easily imagine a situation where things would *necessarily* have no weight, relatively to each other, though each would have its usual weight, looked at by itself.
The Earl: Some desperate paradox! Tell us how it could be. We shall never guess it.

Arthur: Well, suppose this house, just as it is, placed a few billion miles above a planet, and with nothing else near enough to disturb it: of course, it falls to the planet?
The Earl: Of course — though it might take some centuries to do it.

Lady Muriel: And is five-o'clock-tea to be going on all the while?
Arthur: That, and other things. The inhabitants would live their lives, grow up and die, and still the house would be falling, falling, falling! But now as to the relative weight of things. Nothing can be *heavy*, you know, except by *trying* to fall, and being prevented from doing so. You all grant that?

All: Yes.

Arthur: Well, now, if I take this book, and hold it out at arm's length, of course I feel its *weight*. It is trying to fall, and I prevent it. And, if I let go, it falls to the floor. But, if we were all falling together, it couldn't be *trying* to fall any quicker, you know: for, if I let go, what more could it do than fall? And, as my hand would be falling too — at the same rate — it would never leave it, for that would be to get ahead of it in the race. And it could never overtake the falling floor!

Lady Muriel: I see it clearly. But it makes me dizzy to think of such things! How *can* you make us do it?

Myself: There is a more curious idea yet. Suppose a cord fastened to the house, from below, and pulled down by someone on the planet. Then of course the *house* goes faster than its natural rate of falling: but the furniture — with our noble selves — would go on falling at their old pace, and would therefore be left behind.

The Earl: Practically, we should rise to the ceiling. The inevitable result of which would be concussion of brain.

Arthur: To avoid that, let us have the furniture fixed to the floor, and ourselves tied down to the furniture. Then the five-o'clock-tea could go on in peace.

Lady Muriel: With one little drawback! We should take the *cups* down with us: but what about the *tea*?

Arthur: I had forgotten the *tea*. *That*, no doubt, would rise to the ceiling — unless you chose to drink it on the way!

The Earl: Which, I think, is *quite* nonsense enough for one while!

Enough nonsense, indeed! After all these excursions into the world of his alter ego, Lewis Carroll, we now turn our attention to the early life of Charles Dodgson himself.